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Prandtl number effect on external natural convection heat transfer from irregular three-dimensional bodies

A. V. HASSANI and K. G. T. HOLLANDS
Department of Mechanical Engineering, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

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Abstract—A general-purpose expression developed previously for predicting external natural convection heat transfer from three-dimensional bodies of arbitrary shape is extended to cover a wider range of Prandtl numbers: $0.71 \leqslant Pr \leqslant 2000$. The predictions from this expression are compared with the experimental data of several researchers over the range $10^6 < Ra < 10^{13}$ for Prandtl numbers of approximately 10 and 2000. The agreement between the predictions and the experimental data is very good with r.m.s. differences of less than 10% in all cases.

INTRODUCTION

EXTERNAL natural convection heat transfer has been of interest for many years. Many researchers focused on the heat transfer from spheres and long circular cylinders; only a few paid attention to other shapes. Among the latter are Astrauskas [1], Weber *et al.* [2], Chamberlain *et al.* [3], Sparrow and Stretton [4] and Hassani and Hollands [5]. Their combined data covered a Prandtl number range of $0.71 \leqslant Pr \leqslant 2000$. Astrauskas [1] and Weber *et al.* [2] performed natural convection mass transfer measurements by the electrochemical method ($Pr \approx 2000$) for spheres, cones, tetrahedrons, cubes and disks. Chamberlain *et al.* [3] and Hassani and Hollands [5], using air ($Pr \approx 0.71$), covered a variety of shapes such as cubes, spheres, bispheres, oblate and prolate spheroids, short circular cylinders and square and circular disks. Sparrow and Stretton [4] used water ($Pr \approx 6$) and air for their measurements of heat transfer from cubes at different orientations. Except for the data of Weber *et al.* [2], which covers a range of $10^8 < Ra < 10^{12}$, the remaining data are for $Ra < 10^9$.

The prediction of heat transfer from these bodies is for the most part limited to empirical correlations suggested by researchers. These correlations are themselves restricted to certain geometries or range of Rayleigh numbers. Recently, however, Hassani and Hollands [5], by simplifying the method of Raithby and Hollands [6, 7], introduced a general-purpose expression for predicting heat transfer from three-dimensional bodies of arbitrary shape. They showed that a good agreement between their expression and the available experimental data of various shapes existed for $Pr \approx 0.71$. The present work extends the applicability of their expression to Prandtl numbers other than 0.71, namely $0.71 \leqslant Pr \leqslant 2000$ and compares the resulting predictions with the available experimental data having $Pr > 0.71$.

PREDICTIONS

The method for predicting external natural convection heat transfer from three-dimensional shapes outlined in ref. [5] resulted in

$$Nu_{\sqrt{A}} = [[(\bar{C}_l\, Ra_H^{1/4})^m + (\hat{C}_t\, Ra_H^{1/3})^m]^{n/m} + (Nu_{c,\sqrt{A}})^n]^{1/n} \quad (1)$$

where the exponents n and m are given by

$$n = \left[1.26 - \frac{(2-\sqrt{A/L_m})}{9\sqrt{(1-4.79v^{2/3}/A)}}, 1.0\right]_{\max} \quad (2)$$

and

$$m = 2.5 + 12.0\exp(-13|\hat{C}_t\, Ra_H^{1/12} - 0.5|) \quad (3)$$

(see Nomenclature for meaning of symbols). The coefficients $\bar{C}_l$ and $\hat{C}_t$ of equation (1) are functions of Prandtl number. The expression for the mean laminar coefficient $\bar{C}_l$ is

$$\bar{C}_l = \frac{0.671}{[1+(0.492/Pr)^{9/16}]^{4/9}}. \quad (4)$$

The second coefficient $\hat{C}_t$ depends on the body shape as well as Prandtl number. It is defined as

$$\hat{C}_t = \bar{C}_t \frac{\sqrt{A}}{H} \quad (5)$$

where $\bar{C}_t$ is the mean turbulent coefficient given by

$$\bar{C}_t = \frac{1}{A}\int_A C_t(\phi)\, \mathrm{d}A \quad (6)$$

where $C_t(\phi)$ is the local turbulent coefficient, given as a function of the local surface angle ϕ in ref. [7].

To provide a simple method for calculating $\bar{C}_t$, Hassani and Hollands [5] approximated the function $C_t(\phi)$ by a function judiciously chosen so that the integral (equation (6)) required for the evaluation of

NOMENCLATURE

a	coefficient defined by equation (9)
A	heat transfer surface area of body [m^2]
A_h	horizontal downward facing surface of a heated body, or horizontal upward facing surface of a cooled body [m^2]
b	coefficient defined by equation (10)
$\bar{C}_l$	function of Pr given by equation (4)
C_t	local turbulent coefficient, a function of Pr and ϕ (see ref. [7])
$\bar{C}_t$	average value of C_t over body, equation (6)
$\hat{C}_t$	$\bar{C}_t\sqrt{A}/H$
e	coefficient defined by equation (11)
H	characteristic length, $(z_f\bar{P}^2)^{1/3}$ [m]
k	thermal conductivity of fluid [$W\,m^{-1}\,K^{-1}$]
l	side dimension of tetrahedron or cube [m]
L_m	longest straight line passing through the body [m]
m	exponent given by equation (3)
n	exponent given as a function of body shape, equation (2)
$Nu_{\sqrt{A}}$	Nusselt number based on $\sqrt{A}$, $Q\sqrt{A}/A\Delta Tk$
$Nu_{c,\sqrt{A}}$	conduction Nusselt number based on $\sqrt{A}$, i.e. limit of $Nu_{\sqrt{A}}$ as $Ra \to 0$
$\bar{P}$	perimeter averaged over the total height of the body, $(1/z_f)\int_0^{z_f} P(z)\,dz$ [m]
$P(z)$	perimeter of planform of body at elevation z [m]
Pr	Prandtl number of fluid
Pr^*	function of Prandtl number given by equation (12)
Q	convective heat transfer from body [W]
R	radius of sphere, base of cone or circular disk [m]
Ra_H	Rayleigh number based on H, $g\beta\Delta TH^3/\nu\alpha$
ΔT	temperature difference between body and fluid far from body [K]
v	volume of body [m^3]
z_f	total height of body [m].

Greek symbols

α	thermal diffusivity of fluid [$m^2\,s^{-1}$]
β	volumetric coefficient of thermal expansion for fluid [K^{-1}]
ν	kinematic viscosity [$m^2\,s^{-1}$]
ϕ	angle measured between the vertical direction and the local point on the surface of the body.

$\bar{C}_t$ would be tractable and yield simple results. This approximate function for C_t is

$$C_t = a+b\sin(\phi)+e\cos(\phi), \quad \text{for } -\pi/2 < \phi \leqslant \pi/2 \tag{7}$$

$$C_t = 0, \quad \text{for } \phi = -\pi/2. \tag{8}$$

The coefficients a, b and e are to be chosen to best fit the curves given in ref. [7] for a specified Prandtl number. Hassani and Hollands [5] evaluated a, b and e for $Pr \approx 0.71$ and obtained: $a = 0.098$, $b = 0.033$ and $e = 0.008$.

To establish a more general relationship between the coefficients a, b and e and Pr, the above procedure was, in the present study, repeated for Prandtl numbers ranging from 0.71 to 2000, using the fitting procedure detailed in ref. [5] for $Pr \approx 0.71$. The coefficients a, b and e obtained in this way are listed in Table 1. The following equations approximate the values in the table and permit easy interpolation:

$$a = 0.0972 \tag{9}$$

$$b = -0.06\,Pr^*+0.0815 \tag{10}$$

and

$$e = 0.08\,Pr^*-0.0548-6\times10^{-6}\,Pr \tag{11}$$

where Pr^* is given by

$$Pr^* = \frac{Pr^{0.22}}{(1+0.61\,Pr^{0.81})^{0.42}}. \tag{12}$$

Integrating equation (6) results in an expression for $\bar{C}_t$ in terms of purely geometric properties of the body, and of a, b and e [5]

$$\bar{C}_t = a+(b-a)\frac{A_h}{A}+e\frac{z_f\bar{P}}{A} \tag{13}$$

where A_h is the horizontal downward facing surface area for a heated body or horizontal upward facing surface area for a cooled body. Using equation (1) with proper coefficients obtained from equations (4) and (9)–(13), one can predict the heat transfer from

Table 1. Coefficients of equation (13)

Pr	a	b	e
0.71	0.0980	0.0330	0.0080
1.0	0.0980	0.0314	0.0108
2.0	0.0981	0.0293	0.0147
4.0	0.0981	0.0288	0.0159
6.0	0.0981	0.0289	0.0154
50	0.0973	0.0354	0.0042
100	0.0970	0.0384	−0.0003
500	0.0967	0.0482	−0.0107
1000	0.0965	0.0518	−0.0159
2000	0.0964	0.0535	−0.0255

Table 2. Coefficients of equation (1)

Body shape	Pr	$\bar{C}_l$	$\hat{C}_t$	$Nu_{c,\sqrt{A}}$	n	H
sphere	0.71	0.515	0.101	3.545	1.0	3.652 R
sphere	5.8	0.608	0.107	3.545	1.0	3.652 R
sphere	6.5	0.611	0.107	3.545	1.0	3.652 R
sphere	14.0	0.630	0.105	3.545	1.0	3.652 R
sphere	1800	0.668	0.076	3.545	1.0	3.652 R
cube, face up	6.0	0.609	0.094	3.388	1.11	2.520 l
cube, face up	2000	0.668	0.068	3.388	1.11	2.520 l
cube, edge up	0.71	0.515	0.100	3.388	1.11	2.545 l
cube, edge up	6.0	0.609	0.106	3.388	1.11	2.545 l
cube, edge up	2000	0.668	0.071	3.388	1.11	2.545 l
cube, vertex up	6.0	0.609	0.107	3.388	1.11	2.548 l
cube, vertex up	2000	0.668	0.069	3.388	1.11	2.548 l
cone, apex up	2000	0.668	0.077	3.510	1.04	3.327 R
cone, apex down	2000	0.668	0.088	3.510	1.04	3.327 R
tetrahedron (pointing up)	2000	0.668	0.071	3.454	1.18	1.225 l
tetrahedron (pointing down)	2000	0.668	0.082	3.454	1.18	1.225 l
circular disk (horizontal)	2000	0.668	0.112	3.330	1.17	1.740 R

a three-dimensional shape in an infinite medium for a Prandtl number range of $0.71 \leqslant Pr \leqslant 2000$.

COMPARISON WITH EXPERIMENTAL DATA

The predicted results of equation (1) were compared with the experimental data of Weber *et al.* [2] and Astrauskas [1] for $Pr \approx 2000$ and various geometries, such as cubes, tetrahedrons, cones and circular disks. The coefficients for these geometries at $Pr \approx 2000$ are listed in Table 2 and the results are plotted in Figs. 1–4. The agreement is very good—r.m.s. differences of less than 10% are observed in all cases.

The experimental data of Sparrow and Stretton [4] for various orientations of the cube for $Pr \approx 6.0$, as well as the predictions of equation (1), are shown in Fig. 4. The comparison between the predictions and the data show r.m.s. differences of less than 7% for all orientations. The coefficients of equation (1) for these orientations and $Pr \approx 6$ are listed in Table 2.

At high Rayleigh numbers where the turbulence becomes important, the dominant term in equation (1) is $\hat{C}_t\, Ra_H^{1/3}$, in which $\hat{C}_t$ is a function of Prandtl number. For a given shape this latter function is found to have the following characteristics, illustrated in Table 2:

$$\hat{C}_t\,(Pr \approx 2000) < \hat{C}_t\,(Pr \approx 0.71) < \hat{C}_t\,(Pr \approx 6).$$

For example for the cube (edge up), $\hat{C}_t$ for $Pr \approx 6$ is approximately 6% higher than the $\hat{C}_t$ for $Pr \approx 0.71$ and almost 30% higher than the $\hat{C}_t$ for $Pr \approx 2000$. A similar trend can be observed for $\hat{C}_t$ for a sphere. The magnitude of the effect of the Prandtl number, shows up in the predictions shown in Figs. 5 and 6, and to some extent, in the experimental data. Figure 5 shows the experimental data of several workers for a sphere for a Prandtl number range of $0.71 \leqslant Pr \leqslant 1800$. The data of Schmidt [8] using alcohol and water ($5.8 < Pr < 14$) show very good agreement with predictions of equation (1), the r.m.s. difference between the experimental data and the predictions being less

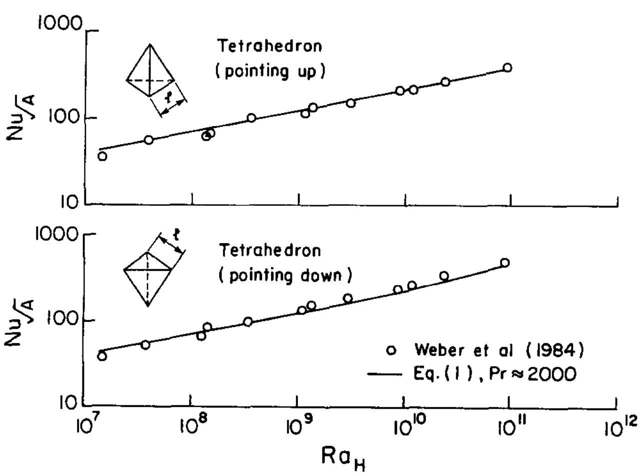

FIG. 1. Comparison of the experimental data and the predictions for a tetrahedron.

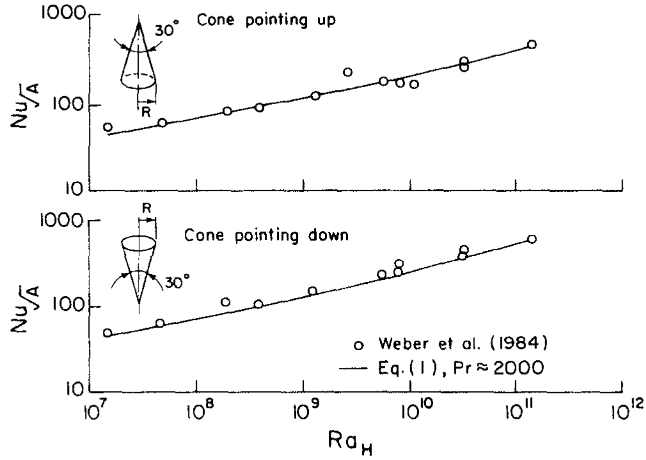

FIG. 2. Comparison of the experimental data and the predictions for a cone.

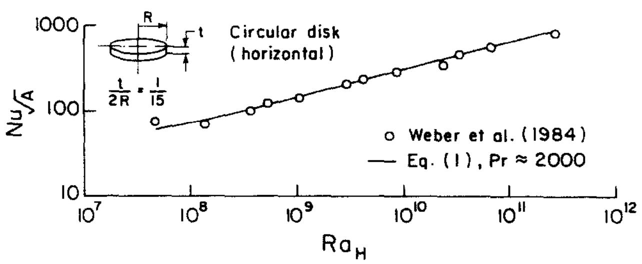

FIG. 3. Comparison of the experimental data and the predictions for a horizontal circular disk.

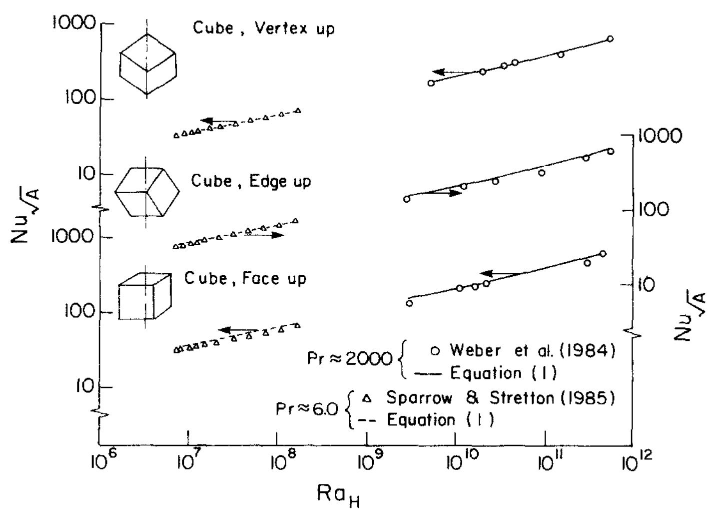

FIG. 4. Comparison of the experimental data and the predictions for a cube in three different orientations.

than 8%. The data of Schutz [10] for $Pr \approx 1800$ is in good agreement with equation (1) with an r.m.s. difference of less than 7%. But the experimental data of Kutateladze [9] for $Pr \approx 0.71$ are generally higher than the predicted results with an r.m.s. difference of 15%. So the experimental data do not unambiguously show the expected trend with Pr; Kutateladze's data for air almost coincides with Schmidt's data for water. It may be that the error, involved in the experiments of Kutateladze [9] causes his data to be consistently high (see Churchill [11] for a detailed study for spheres). Figure 6 shows the experimental data of Weber *et al.* [2], Sparrow and Stretton [4], and Chamberlain *et al.* [3] for the cube (edge up) for different Prandtl numbers. The agreement between the experimental data and the predictions of equation (1) is very good for all cases, with r.m.s. differences of less than 7%. Unfortunately, however, the exper-

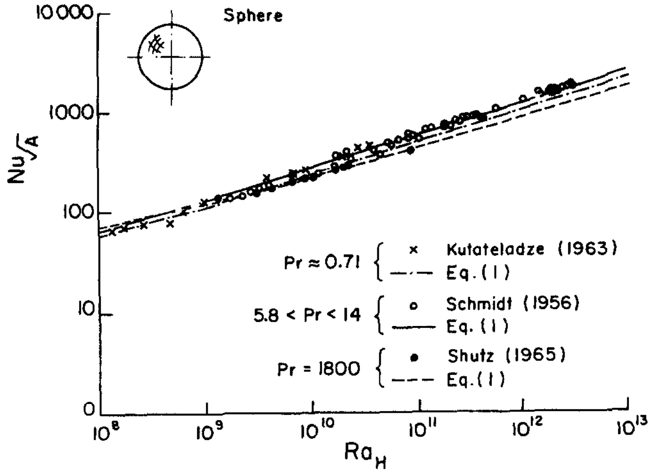

FIG. 5. Comparison of the experimental data and the predictions for a sphere.

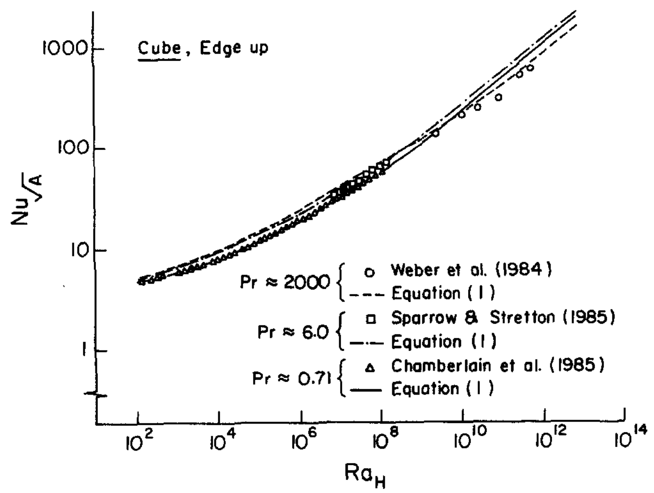

FIG. 6. The effect of Prandtl number on heat transfer from a cube with its edge up.

imental ranges of the Rayleigh number are not sufficient to properly evaluate the agreement with the expected trend.

CONCLUSIONS

The equation developed by Hassani and Hollands [5] from the earlier work of Raithby and Hollands has been extended to cover a wide range of Prandtl numbers: $0.71 \leqslant Pr \leqslant 2000$. The predictions of this correlation had already been verified by Hassani and Hollands for various shapes over a Rayleigh number range of $10 < Ra < 10^9$ for $Pr \approx 0.71$. This work has compared the predictions with the experimental data of Astrauskas [1], Weber *et al.* [2], Sparrow and Stretton [4], and Schmidt [8] for various geometries over $10^6 < Ra < 10^{13}$ for $Pr \approx 2000$ and $5.8 < Pr < 14$. The agreement between the predictions and the experimental data is very good and r.m.s. differences of less than 10% are observed in all cases. A need for more experimental data at intermediate Prandtl numbers and high Rayleigh numbers is indicated.

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EFFET DU NOMBRE DE PRANDTL SUR LA CONVECTION THERMIQUE NATURELLE EXTERNE AUTOUR DE CORPS TRIDIMENSIONNELS IRREGULIERS

Résumé—Une formule développée antérieurement pour prédire le transfert de chaleur par convection naturelle autour de corps tridimensionnels de forme arbitraire est étendue pour couvrir un plus large domaine de nombre de Prandtl $0{,}71 \leqslant Pr \leqslant 2000$. Les prédictions de ces expressions sont comparées aux données expérimentales de plusieurs auteurs dans le domaine $10^6 < Ra < 10^{13}$, pour des nombres de Prandtl de 10 à 2000. L'accord est très bon avec des moyennes quadratiques de différences inférieures à 10% dans tous les cas.

DER EINFLUSS DER PRANDTL-ZAHL AUF DEN WÄRMEÜBERGANG AN DER AUSSENSEITE EINES UNREGELMÄSSIGEN DREIDIMENSIONALEN KÖRPERS

Zusammenfassung—In der vorliegenden Arbeit wird ein kürzlich entwickelter allgemeiner Ausdruck für die Berechnung des Wärmeübergangs durch natürliche Konvektion an der Außenseite eines dreidimensionalen Körpers beliebiger Form so erweitert, daß nun Prandtl-Zahlen zwischen 0,71 und 2000 abgedeckt werden können. Die Ergebnisse derartiger Berechnungen werden mit experimentellen Daten unterschiedlicher Herkunft im Bereich $10^6 < Ra < 10^{13}$ und $10 \leqslant Pr \leqslant 2000$ verglichen. Die Übereinstimmung zwischen Meß- und Rechenwerten ist sehr gut, die r.m.s.-Abweichung ist in allen Fällen kleiner als 10%.

ВЛИЯНИЕ ЧИСЛА ПРАНДТЛЯ НА ВНЕШНИЙ СВОБОДНОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС ОТ ТРЕХМЕРНЫХ ТЕЛ НЕПРАВИЛЬНОЙ ФОРМЫ

Аннотация—Предложенное ранее выражение для расчета внешнего свободноконвективного теплопереноса от трехмерных тел произвольной формы обобщается на более широкий диапазон чисел Прандтля $0{,}71 \leqslant Pr \leqslant 2000$. Полученные с помощью этого выражения результаты сравниваются с экспериментальными данными нескольких исследователей в интервале чисел Рэлея от 10^6 до 10^{13}, соответствующем приблизительно изменению числа Прандтля от 10 до 2000. Среднеквадратичные отклонения расчетных значений от экспериментальных во всех случаях составляют менее 10%, что указывает на очень хорошее согласие теории с экспериментом.